Lesson 18: Integral Process Characteristics

ET 438a Automatic Control Systems Technology

Learning Objectives

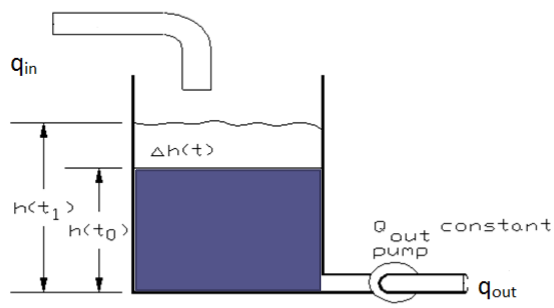
After this series of presentations you will be able to:

- Describe typical process models found in control systems.
- Write mathematical formulas for process models
- Compute the parameters of process models.
- Identify the Bode plots of typical process models.
- Identify the time response of typical process models.

Integral Processes

Integral Process Characteristic - single energy storage element (capacitance) and no losses.

Example: Tank with constant output flow. (Pumped outflow)



For $q_{in} > q_{out}$ h increases

For $q_{out} > q_{in}$ h decreases

Pump maintains constant outflow regardless of tank height

Integral Processes

Mathematical Descriptions

Time domain equation: $$h(t_1) - h(t_0) = \frac{1}{T_i}\int_{t_0}^{t_1}(q_{in}(t) - q_{iout}(t))\,dt$$

Transfer function: $$\frac{H(s)}{Q_{in}(s)} = \frac{1}{T_i \cdot s}$$

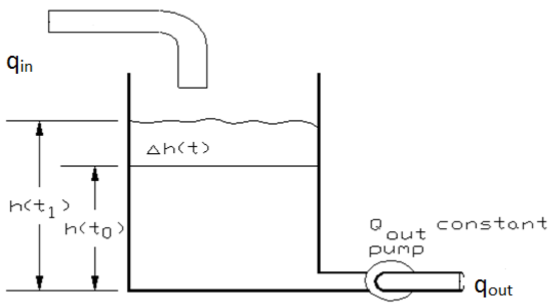
Integral Constant: $$T_i = A \cdot \left[\frac{FS_{out}}{FS_{in}}\right]$$

Where:

$h(t_o)$ = normalized output at t_o as percentage of FS_{out}
$h(t_1)$ = normalized output at t_1 as percentage of FS_{out}
FS_{in} = full-scale input flow q_{in}
FS_{out} = full-scale output height h
T_i = integral action time constant
A = cross-sectional tank area (m^2)
$q_{in}(t)$ normalize input flow as % of FS_{in}.
$q_{out}(t)$ normalize output flow as % of FS_{in}.

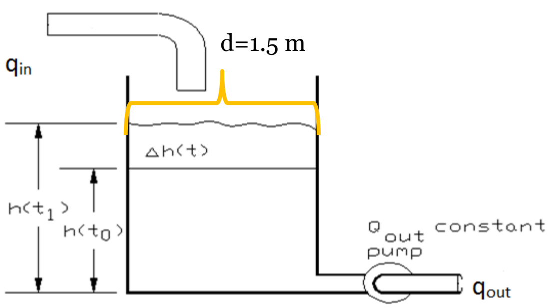
Integral Processes

Example 18-1: Find the for the tank/pump system shown: a) the transfer function if the tank diameter is 1.5 m and the height is 4 m. Full scale q_{in} = 0.01 m³/s and full scale height is 4 m. b) determine the time domain equation and tank level after 100 seconds from t_o.



$h(t_o)$= 22.5% FS
q_{out} = 60% FS
q_{in} = 80% FS

Example 18-1 Solution (1)



$$A = \frac{\pi d^2}{4}$$

$$A = \frac{\pi (1.5m)^2}{4} = 1.77 m^2$$

$h(t_o)$ = 22.5% FS
q_{out} = 60% FS
q_{in} = 80% FS

FS_{out} = 4 m
$FS_{in} = q_{in} = 0.01\ m^3/s$

Example 18-1 Solution (2)

Convert flows and initial height into actual values from the given percentages

$$FS_{out}\left[\frac{\%}{100}\right] = h_0 \qquad FS_{in}\left[\frac{\%}{100}\right] = q_{out}$$

$$4\,m\left[\frac{22.5\%}{100\%}\right] = h_0 \qquad 0.01\,m^3/s\left[\frac{60\%}{100}\right] = 0.006\,m^3/s = q_{out}$$

$$0.9\,m = h_0$$

$$FS_{in}\left[\frac{80\%}{100}\right] = 0.008\,m^3/s = q_{in}$$

$$T_{in} = A\left[\frac{FS_{out}}{FS_{in}}\right] = \frac{1.77\,m^2(4\,m)}{0.01\,m^3/s} = 703\,s \qquad \frac{H(s)}{Q_{in}(s)} = \frac{1}{T_{in}s} = \boxed{\frac{1}{703\,s}}$$

Ans a

Example 18-1 Solution (3)

Now find the answer to part b. Use the equation from slide 4 for the integral

$$h(t_1) - h(t_0) = \frac{1}{T_{in}}\int_{t_0}^{t_1}(q_{in} - q_{out})\,dt$$

Solve for final height

$$h(t_1) = \frac{1}{T_{in}}\int_{t_0}^{t_1}(q_{in} - q_{out})\,dt + h(t_0)$$

$t_1 = 100$ sec

$h(t_0) = 22.5\%$ $(0.9\,m)$

Assume $t_0 = 0$ start time

Example 18-1 Solution (4)

Calculation can be done using actual flow and height values of percentages. Use percentages in this case.

$$q_{in} = 80\% \ (0.008\ m^3/s) \qquad q_{out} = 60\% \ (0.006\ m^3/s)$$

$$T_{in} = 703\ s$$

Place values in this equation and integrate

$$h(t_1) = \frac{1}{T_{in}} \int_{t_0}^{t_1} (q_{in} - q_{out})\,dt + h(t_0)$$

$$h(100) = \frac{1}{703} \int_{0}^{100} (80-60)\,dt + 22.5\%$$

Example 18-1 Solution (5)

Integrate the difference between the flows.

$$h(100) = \frac{1}{703} \int_{0}^{100} (80-60)\,dt + 22.5\%$$

$$h(100) = \frac{1}{703}\left[20t\Big|_0^{100}\right] + 22.5\%$$

$$h(100) = \frac{1}{703}\left[20(100)\right] + 22.5\%$$

$$h(100) = 2.85\% + 22.5\% = 25.34\%$$

Ans

Convert h% to meters: $FS_{out}\left[\frac{\%}{100}\right] = h \qquad 4m\left[\frac{25.34\%}{100}\right] = h \qquad \boxed{1.014\ m = h}$

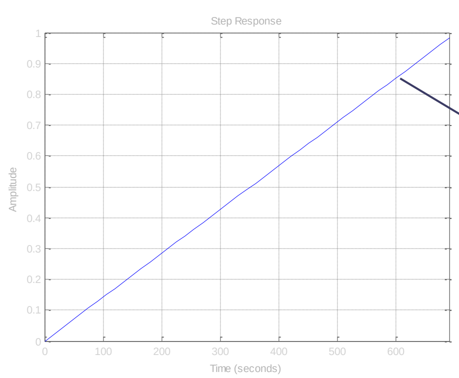
Step Response and Bode Plots of The Integral Process

MatLAB Code

```
%close all previous figures and clear all
variables
close all;
clear all;
%input the integral time constant
Ti=input('Enter the integral time constant: ');
% construct and display the system
sys=tf(1,[Ti 0]);
sys
%Plot the frequency response
bode(sys);
Grid on;
%Construct a new figure and plot the time
response
figure;
%define a range of time
t=(0:15:Ti);  %(start time, Stop time, final
time)
% use this range to generate a step response
step(sys,t);
Grid on;
```


Integral Process Time Response

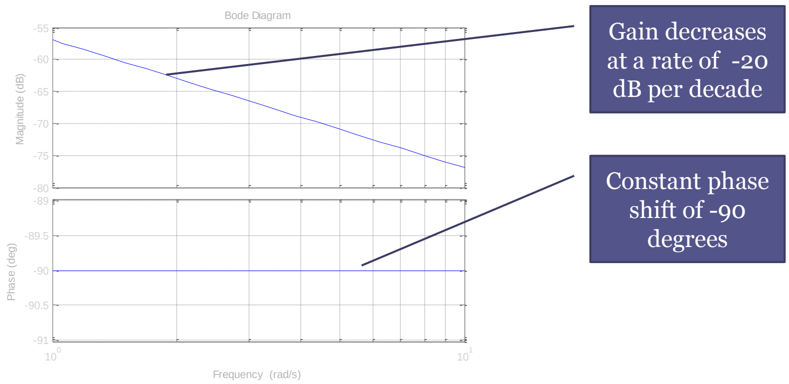
Integral response to step input (0-1) transition



Output reaches final value at value of T_i

Integral Process Frequency Response

Integrator has infinite gain to dc signals with decreasing response as frequency increases.



End Lesson 18: Process Characteristics-Integral Process

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